

Λ -renormalized Einstein-Schrödinger theory: an alternative to Einstein-Maxwell theory

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Λ -renormalized Einstein-Schrödinger (LRES) theory

- Vacuum general relativity can be derived from a Palatini Lagrangian density,

$$\mathcal{L}(\Gamma_{\rho\tau}^\lambda, g_{\rho\tau}) = -\frac{1}{16\pi} [\sqrt{-g} g^{\mu\nu} R_{\nu\mu}(\Gamma) + 2\Lambda_b \sqrt{-g}]. \quad (1)$$

- Einstein-Schrödinger theory uses $\hat{\Gamma}_{\mu\nu}^\alpha$ and $N_{\mu\nu}$ with no symmetry properties,

$$\mathcal{L}(\hat{\Gamma}_{\rho\tau}^\lambda, N_{\rho\tau}) = -\frac{1}{16\pi} [\sqrt{-N} N^{-1\mu\nu} R_{\nu\mu}(\hat{\Gamma}) + 2\Lambda_b \sqrt{-N}], \quad N = \det(N_{\mu\nu}) \quad (2)$$

- LRES theory includes Λ_z from zero-point fluctuations and allows other fields,

$$\begin{aligned} \mathcal{L}(\hat{\Gamma}_{\rho\tau}^\lambda, N_{\rho\tau}) = & -\frac{1}{16\pi} [\sqrt{-N} N^{-1\mu\nu} R_{\nu\mu}(\hat{\Gamma}) + 2\Lambda_b \sqrt{-N}] \\ & -\frac{1}{16\pi} 2\Lambda_z \sqrt{-g} + \mathcal{L}_m(\psi_e, A_\nu, g_{\mu\nu}, \dots) \end{aligned} \quad (3)$$

where the “bare” $\Lambda_b \approx -\Lambda_z$ so the “physical” $\Lambda = \Lambda_b + \Lambda_z$ matches measurement, and \mathcal{L}_m lacks $F^{\mu\nu}F_{\mu\nu}$ part, and the metric $g_{\mu\nu}$ and potential A_ν are defined by

$$\sqrt{-g} g^{\nu\mu} = \sqrt{-N} N^{-1(\mu\nu)}, \quad A_\nu = \hat{\Gamma}_{[\rho\nu]}^\rho \sqrt{-2} \Lambda_b^{-1/2} / 6, \quad (\text{with } c=G=1). \quad (4)$$

- $\lim_{|\Lambda_z| \rightarrow \infty} \left(\begin{array}{c} \text{LRES} \\ \text{theory} \end{array} \right) = \left(\begin{array}{c} \text{Einstein-Maxwell} \\ \text{theory} \end{array} \right)$ but $\omega_c \sim \frac{1}{l_P} \Rightarrow |\Lambda_z| \sim \omega_c^4 l_P^2 \sim \frac{1}{l_P^2}$.

LRES theory matches measurement as well as Einstein-Maxwell theory

- Reduces to ordinary GR without electromagnetism for symmetric fields.
- Lorentz force equation is identical to that of Einstein-Maxwell theory.
- Extra terms in Einstein and Maxwell equations are $< 10^{-16}$ of usual terms for worst-case $|F_{\mu\nu}|$, $|F_{\mu\nu;\alpha}|$ and $|F_{\mu\nu;\alpha;\beta}|$ accessible to measurement.
- Exact solutions:
 - EM plane-wave solution is identical to that of Einstein-Maxwell theory.
 - Charged solution and Reissner-Nordström sol. have tiny fractional difference: $10^{-76} @ r = Q = M = M_{\odot}$; $10^{-64} @ r = 10^{-17} \text{ cm}, Q = e, M = M_e$.
- Standard tests | fractional difference from Einstein-Maxwell result

Standard tests	fractional difference from Einstein-Maxwell result
periastron advance	$10^{-78} @ Q = M = M_{\odot}, r = 4M$; $10^{-91} @ Q_1 = -Q_2 = e, r = a_0$
deflection of light	$10^{-79} @ Q = M = M_{\odot}, b = 4M$; $10^{-57} @ Q = e, M = M_P, b = a_0$
time delay of light	$10^{-78} @ Q = M = M_{\odot}, b = 4M$; $10^{-56} @ Q = e, M = M_P, b = a_0$
- Other Standard Model fields can be added just like Einstein-Maxwell theory:
 - Energy levels of Hydrogen atom have fractional difference of $< 10^{-50}$.

LRES theory avoids the problems of Einstein-Schrödinger theory

- Matches measurement as well as Einstein-Maxwell theory.
- Definitely predicts a Lorentz force:
 - Usual Lorentz force equation results from divergence of Einstein equations,
 - Lorentz force also results from the EIH method, with $\mathcal{L}_m = 0$.
- Avoids ghosts:
 - With a cutoff frequency $\omega_c \sim 1/l_P$ we have $\Lambda_z \sim -\omega_c^4 l_P^2$ (with $c=G=1$),
 - Ghosts are cut off because they would have $\omega_{ghost} = \sqrt{-2\Lambda_z} \sim \sqrt{2} \omega_c^2 l_P > \omega_c$,
 - If we fully renormalize with $\omega_c \rightarrow \infty$ then $\omega_{ghost} \rightarrow \infty$, meaning no ghost.
- Well motivated:
 - It's a vacuum energy renormalization of Einstein-Schrödinger theory,
 - $\Lambda_z \sqrt{-g}$ term should be expected to occur as a quantization effect,
 - Zero-point fluctuations are essential to QED - they cause the Casimir effect,
 - $\Lambda = \Lambda_b + \Lambda_z$ is similar to mass/charge/field-strength renormalization in QED,
 - $\Lambda_z \sqrt{-g}$ modification has never been considered before.

Why pursue LRES theory if it's so close to Einstein-Maxwell theory?

- It unifies gravitation and electromagnetism in a classical sense.
- Quantization of LRES theory is untried approach to quantization of gravity:
 - LRES theory gets much different than Einstein-Maxwell theory as $k \rightarrow 1/l_P$,
 - This could possibly fix some infinities which spoil the quantization of GR.
- LRES theory suggests untried approaches to a complete unified field theory:
 - Higher dimensions, but with LRES theory instead of vacuum GR?
 - Non-abelian fields, but with LRES theory instead of Einstein-Maxwell?
- We still don't have a unified field theory, 50 years after Einstein:
 - Standard Model: excludes gravity, 25 parameters, not very "beautiful",
 - String theory: background dependent, spin-2 particle \Rightarrow GR?,
 $\sim 10^{500}$ versions, problems accounting for $\Lambda > 0$ and broken symmetry.
- New ideas are needed - many approaches should be pursued, not just one.

The Lagrangian Density Again

- A_ν and $F_{\mu\nu}$ are defined by (with $c=G=1$)

$$A_\nu = \widehat{\Gamma}_{[\rho\nu]}^\rho \sqrt{-2} \Lambda_b^{-1/2} / 6, \quad (5)$$

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}. \quad (6)$$

- $\widehat{\Gamma}_{\nu\mu}^\alpha$ can be decomposed into $\tilde{\Gamma}_{\nu\mu}^\alpha$ with the symmetry $\tilde{\Gamma}_{\nu\alpha}^\alpha = \tilde{\Gamma}_{\alpha\nu}^\alpha$, and A_ν ,

$$\tilde{\Gamma}_{\nu\mu}^\alpha = \widehat{\Gamma}_{\nu\mu}^\alpha - 2\delta_\nu^\alpha \widehat{\Gamma}_{[\rho\mu]}^\rho / 3 \quad (7)$$

$$\Rightarrow \widehat{\Gamma}_{\nu\mu}^\alpha = \tilde{\Gamma}_{\nu\mu}^\alpha - 2\delta_\nu^\alpha A_\mu \sqrt{-2} \Lambda_b^{1/2} \quad (8)$$

- The Lagrangian density (3) in terms of A_μ , $\tilde{\Gamma}_{\nu\mu}^\alpha$ and $\tilde{R}_{\nu\mu} = R_{\nu\mu}(\tilde{\Gamma})$ is,

$$\begin{aligned} \mathcal{L}(\widehat{\Gamma}_{\rho\tau}^\lambda, N_{\rho\tau}) = & -\frac{1}{16\pi} \left[\sqrt{-N} N^{-1\mu\nu} (\tilde{R}_{\nu\mu} + 2A_{[\nu,\mu]} \sqrt{-2} \Lambda_b^{1/2}) + 2\Lambda_b \sqrt{-N} \right] \\ & -\frac{1}{16\pi} 2\Lambda_z \sqrt{-g} + \mathcal{L}_m(\psi_e, A_\nu, g_{\mu\nu}, \dots) \end{aligned} \quad (9)$$

- $\tilde{R}_{\nu\mu}$ is actually the ‘‘Hermitianized Ricci tensor’’ with $\tilde{R}_{\mu\nu}(\tilde{\Gamma}^T) = \tilde{R}_{\nu\mu}(\tilde{\Gamma})$,

$$\tilde{R}_{\nu\mu} = \tilde{R}_{\nu\mu}^{(\text{usual})} + \frac{1}{2} \tilde{R}_{\alpha\mu\nu}^{(\text{usual})} = \tilde{\Gamma}_{\nu\mu,\alpha}^\alpha - \tilde{\Gamma}_{(\nu|\alpha,|\mu)}^\alpha + \tilde{\Gamma}_{\nu\mu}^\rho \tilde{\Gamma}_{\rho\alpha}^\alpha - \tilde{\Gamma}_{\nu\alpha}^\rho \tilde{\Gamma}_{\rho\mu}^\alpha. \quad (10)$$

The Einstein Equations

- $g_{\mu\nu}$ and $f_{\mu\nu}$ are defined by (with $c=G=1$)

$$\sqrt{-g} g^{\nu\mu} = \sqrt{-N} N^{-1(\mu\nu)}, \quad (11)$$

$$\sqrt{-g} f^{\nu\mu} = \sqrt{-N} N^{-1[\mu\nu]} \Lambda_b^{1/2} / \sqrt{-2}. \quad (12)$$

Inverting these definitions gives (after some effort)

$$N_{(\nu\mu)} = g_{\nu\mu} - 2 \left(f_{\nu}^{\alpha} f_{\alpha\mu} - \frac{1}{4} g_{\nu\mu} f^{\rho\alpha} f_{\alpha\rho} \right) \Lambda_b^{-1} + \mathcal{O}(\Lambda_b^{-2}), \quad (13)$$

$$N_{[\nu\mu]} = f_{\nu\mu} \sqrt{-2} \Lambda_b^{-1/2} + \mathcal{O}(\Lambda_b^{-3/2}). \quad (14)$$

- $f_{\mu\nu} \approx F_{\mu\nu}$ comes from $\delta\mathcal{L}/\delta(\sqrt{-N} N^{-1[\mu\nu]}) = 0$ and $\tilde{R}_{[\nu\mu]} = \mathcal{O}(\Lambda_b^{-1/2})$ from (28),

$$N_{[\nu\mu]} = 2A_{[\mu,\nu]} \sqrt{-2} \Lambda_b^{-1/2} - \tilde{R}_{[\nu\mu]} \Lambda_b^{-1}, \quad (15)$$

$$\Rightarrow f_{\nu\mu} = A_{\mu,\nu} - A_{\nu,\mu} + \mathcal{O}(\Lambda_b^{-1}). \quad (16)$$

- Einstein equations come from $\delta\mathcal{L}/\delta(\sqrt{-N} N^{-1(\mu\nu)}) = 0$,

$$\tilde{R}_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} \tilde{R}^{\rho}_{\rho} = 8\pi T_{\nu\mu} - \Lambda_b \left(N_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} N^{\rho}_{\rho} \right) + \Lambda_z g_{\nu\mu} \quad (17)$$

$$= 8\pi T_{\nu\mu} + 2 \left(f_{\nu}^{\alpha} f_{\alpha\mu} - \frac{1}{4} g_{\nu\mu} f^{\rho\alpha} f_{\alpha\rho} \right) + \Lambda g_{\nu\mu} + \mathcal{O}(\Lambda_b^{-1}). \quad (18)$$

Maxwell's Equations

- Maxwell's equations come from $\delta\mathcal{L}/\delta A_\tau = 0$ and $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$,

$$f^{\nu\tau}{}_{;\nu} = 4\pi j^\tau, \quad (19)$$

$$F_{[\mu\nu,\alpha]} = 0, \quad (20)$$

where $f_{\mu\nu} \approx F_{\mu\nu}$ and

$$j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta\mathcal{L}_m}{\delta A_\tau}. \quad (21)$$

- \mathcal{L}_m may contain other fields just like Einstein-Maxwell theory,

$$j^\tau = Q\bar{\psi}_e \gamma^\tau \psi_e \quad \text{for spin } -1/2, \quad (22)$$

$$j^\tau = \rho u^\tau \quad \text{for classical hydrodynamics.} \quad (23)$$

The Connection Equations

- Relation between $\tilde{\Gamma}_{\mu\nu}^\alpha$ and $N_{\mu\nu}$ like $(\sqrt{-g}g^{\tau\rho})_{;\beta} = 0$ comes from $\delta\mathcal{L}/\delta\tilde{\Gamma}_{\tau\rho}^\beta = 0$,

$$\begin{aligned}
 (\sqrt{-N}N^{-1\rho\tau})_{;\beta} + \tilde{\Gamma}_{\nu\beta}^\tau\sqrt{-N}N^{-1\rho\nu} + \tilde{\Gamma}_{\beta\nu}^\rho\sqrt{-N}N^{-1\nu\tau} - \tilde{\Gamma}_{\beta\alpha}^\alpha\sqrt{-N}N^{-1\rho\tau} \\
 = \frac{8\pi}{3}\sqrt{-g}j^{[\rho\delta\tau]}_{\beta}\sqrt{-2}\Lambda_b^{-1/2}. \quad (24)
 \end{aligned}$$

- Solving these equations gives

$$\tilde{\Gamma}_{(\nu\mu)}^\alpha = \frac{1}{2}g^{\alpha\rho}(g_{\mu\rho,\nu} + g_{\rho\nu,\mu} - g_{\nu\mu,\rho}) + \mathcal{O}(\Lambda_b^{-1}), \quad (25)$$

$$\tilde{\Gamma}_{[\nu\mu]}^\alpha = \mathcal{O}(\Lambda_b^{-1/2}), \quad (26)$$

$$\tilde{R}_{(\nu\mu)} = R_{\nu\mu} + (\text{terms like } f^{\alpha\tau}f_{\tau(\mu;\nu);\alpha}\Lambda_b^{-1} \text{ and } f^\rho_{\mu;\alpha}f^\alpha_{\nu;\rho}\Lambda_b^{-1}), \quad (27)$$

$$\tilde{R}_{[\nu\mu]} = (\text{terms like } f_{[\mu\nu,\tau];\tau}\Lambda_b^{-1/2}, f^\tau_{[\mu;[\nu];\tau]}\Lambda_b^{-1/2} \text{ and } j_{[\nu,\mu]}\Lambda_b^{-1/2}). \quad (28)$$

$\Rightarrow \tilde{R}_{(\nu\mu)} \approx R_{\nu\mu}$ and $f_{\nu\mu} \approx F_{\nu\mu}$ with fractional differences $< 10^{-16}$ for worst-case $|f_{\mu\nu}|, |f_{\mu\nu;\alpha}|, |f_{\mu\nu;\alpha;\beta}|$ accessible to measurement (e.g. $10^{20}eV, 10^{34}Hz$ γ -rays).

The Generalized Contracted Bianchi Identity

- A generalized contracted Bianchi identity results from (24),

$$(\sqrt{-N}N^{-1\sigma\nu}\tilde{R}_{\nu\lambda} + \sqrt{-N}N^{-1\nu\sigma}\tilde{R}_{\lambda\nu}),_{\sigma} - \sqrt{-N}N^{-1\sigma\nu}\tilde{R}_{\nu\sigma;\lambda} = 0. \quad (29)$$

- It may also be written in the manifestly covariant form,

$$(\sqrt{-N}N^{-1\sigma\nu}\tilde{R}_{\nu\lambda} + \sqrt{-N}N^{-1\nu\sigma}\tilde{R}_{\lambda\nu});_{\sigma} - \sqrt{-N}N^{-1\sigma\nu}\tilde{R}_{\nu\sigma;\lambda} = 0, \quad (30)$$

- Or in a third form,

$$\tilde{G}^{\sigma}{}_{\lambda;\sigma} = \left(\frac{3}{2}f^{\sigma\nu}\tilde{R}_{[\sigma\nu,\lambda]} + 4\pi j^{\nu}\tilde{R}_{[\nu\lambda]} \right) \sqrt{-2}\Lambda_b^{-1/2}, \quad (31)$$

where

$$\tilde{G}_{\nu\mu} = \tilde{R}_{(\nu\mu)} - \frac{1}{2}g_{\nu\mu}\tilde{R}_{\rho}{}^{\rho}. \quad (32)$$

- The usual contracted Bianchi identity $2(\sqrt{-g}g^{\sigma\nu}R_{\nu\lambda}),_{\sigma} - \sqrt{-g}g^{\sigma\nu}R_{\nu\sigma;\lambda} = 0$, or $G^{\sigma}{}_{\lambda;\sigma} = 0$ is also valid.

The Lorentz Force Equation

- Lorentz force equation comes from divergence of the Einstein equations (17)

$$T_{\mu;\nu}^{\nu} = F_{\mu\nu}j^{\nu} \quad (33)$$

where

$$j^{\tau} = \frac{-1}{\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta A_{\tau}}, \quad (34)$$

$$T_{\mu\nu} = S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S_{\alpha}^{\alpha}, \quad (35)$$

$$S_{\mu\nu} = \frac{2 \delta \mathcal{L}_m}{\delta(\sqrt{-g} g^{\nu\mu})}. \quad (36)$$

- Here we have used equations (31,19,15) and the following identity which can be derived using only the definitions of $g_{\mu\nu}$ and $f_{\mu\nu}$,

$$\left(N^{(\mu}_{\sigma)} - \frac{1}{2} \delta_{\sigma}^{\mu} N_{\rho}^{\rho} \right)_{;\mu} = \left(\frac{3}{2} f^{\nu\rho} N_{[\nu\rho,\sigma]} + f^{\nu\rho}_{;\nu} N_{[\rho\sigma]} \right) \sqrt{-2} \Lambda_b^{-1/2}. \quad (37)$$

- Covariant derivative “;” is always done using the Christoffel connection formed from the symmetric metric $g_{\mu\nu}$.

An Exact Charged Solution

- This charged solution is very close to the Reissner-Nordström solution,

$$g_{\nu\mu} = \check{c} \begin{pmatrix} a & & & \\ & -1/a\check{c}^2 & & \\ & & -r^2 & \\ & & & -r^2\sin^2\theta \end{pmatrix}, \quad (38)$$

$$f_{\nu\mu} = \frac{1}{\check{c}} \begin{pmatrix} 0 & Q/r^2 & & \\ -Q/r^2 & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \quad (39)$$

$$A_0 = \frac{Q}{r} \left[1 + \frac{M}{\Lambda_b r^3} - \frac{4Q^2}{5\Lambda_b r^4} + \mathcal{O}(\Lambda_b^{-2}) \right], \quad (40)$$

where

$$a = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \left[1 + \frac{Q^2}{10\Lambda_b r^4} + \mathcal{O}(\Lambda_b^{-2}) \right], \quad \check{c} = \sqrt{1 - \frac{2Q^2}{\Lambda_b r^4}}. \quad (41)$$

- Additional terms are tiny for worst-case radii accessible to measurement:
 - $Q^2/\Lambda_b r^4 \sim 10^{-76} \text{ @ } r=Q=M=M_\odot$; $\sim 10^{-64} \text{ @ } r=10^{-17} \text{ cm}, Q=e, M=M_e$,
 - $M/\Lambda_b r^3 \sim 10^{-76} \text{ @ } r=Q=M=M_\odot$; $\sim 10^{-70} \text{ @ } r=10^{-17} \text{ cm}, Q=e, M=M_e$.

Summary of Λ -renormalized Einstein-Schrödinger theory

- $\lim_{|\Lambda_z| \rightarrow \infty} \left(\begin{array}{c} \text{LRES} \\ \text{theory} \end{array} \right) = \left(\begin{array}{c} \text{Einstein-Maxwell} \\ \text{theory} \end{array} \right)$ but $\omega_c \sim \frac{1}{l_P} \Rightarrow |\Lambda_z| \sim \omega_c^4 l_P^2 \sim \frac{1}{l_P^2}$.
- Matches measurement as well as Einstein-Maxwell theory.
- Reduces to ordinary GR without electromagnetism for symmetric fields.
- Other Standard Model fields can be added just like Einstein-Maxwell theory.
- Avoids the problems of the original Einstein-Schrödinger theory.
- Well motivated - it's the ES theory but with a quantization effect.
- Unifies gravitation and electromagnetism in a classical sense.
- Suggests untried approaches to a complete quantized unified field theory.
- For the details see my papers: www.arxiv.org/abs/gr-qc/0310124, www.arxiv.org/abs/gr-qc/0403052, www.arxiv.org/abs/gr-qc/0411016.