

# **A modification of Einstein-Schrödinger theory which closely approximates Einstein-Weinberg-Salam theory**

James A. Shiflett  
Department of Physics  
WUGRAV Gravity Group  
Washington University in St. Louis

shifflet@hbar.wustl.edu

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## Comparison of the Lagrangians

- Einstein-Weinberg-Salam theory can be derived from a Palatini Lagrangian

$$\begin{aligned} \mathcal{L}(\Gamma_{\rho\tau}^\lambda, g_{\rho\tau}, \mathcal{A}_\nu) = & -\frac{1}{16\pi}\sqrt{-g} [g^{\mu\nu} R_{\nu\mu}(\Gamma) + 2\Lambda_b] \\ & + \frac{1}{32\pi}\sqrt{-g} \operatorname{tr}(f_{\rho\alpha} g^{\alpha\mu} g^{\rho\nu} f_{\nu\mu}) + \mathcal{L}_m(g_{\mu\nu}, \mathcal{A}_\nu, \psi_{e\nu}, \phi \cdots), \end{aligned} \quad (1)$$

where  $\mathcal{A}_\nu = I a_\nu + \sigma_i b_\nu^i$  is composed of  $2 \times 2$  Hermitian matrix components and

$$f_{\nu\mu} = 2\mathcal{A}_{[\mu,\nu]} + \frac{i\sqrt{\alpha}}{2l_P \sin\theta_w} [\mathcal{A}_\nu, \mathcal{A}_\mu]. \quad (2)$$

- Non-Abelian LRES theory allows nonsymmetric  $N_{\mu\nu}$  and  $\widehat{\Gamma}_{\rho\tau}^\lambda$  with  $2 \times 2$  Hermitian matrix components, excludes  $\operatorname{tr}(f_{\rho\alpha} g^{\alpha\mu} g^{\rho\nu} f_{\nu\mu})$ , and includes a  $\Lambda_z$ ,

$$\begin{aligned} \mathcal{L}(\widehat{\Gamma}_{\rho\tau}^\alpha, N_{\rho\tau}) = & -\frac{1}{16\pi} N^{1/4} [\operatorname{tr}(N^{-1\mu\nu} \mathcal{R}_{\nu\mu}(\widehat{\Gamma})) + 4\Lambda_b] \\ & - \frac{1}{4\pi} g^{1/4} \Lambda_z + \mathcal{L}_m(g_{\mu\nu}, \mathcal{A}_\nu, \psi_{e\nu}, \phi \cdots), \quad N = \det(N_{\mu\nu}) \end{aligned} \quad (3)$$

where the “bare”  $\Lambda_b \approx -\Lambda_z$  so that  $\Lambda = \Lambda_b + \Lambda_z$  matches measurement, and

$$\mathcal{A}_\nu = \widehat{\Gamma}_{[\nu\sigma]}^\sigma / \sqrt{-18\Lambda_b}, \quad g^{1/4} g^{\mu\nu} = N^{1/4} N^{-1(\mu\nu)}, \quad (g_{\nu\mu} = I g_{\nu\mu} \text{ assumed}). \quad (4)$$

## The field equations

- The electro-weak field tensor  $f^{\nu\mu}$  is defined by

$$g^{1/4} f^{\nu\mu} = iN^{1/4} N^{-1[\nu\mu]} \Lambda_b^{1/2} / \sqrt{2}, \quad (5)$$

- Ampere's law is identical to Weinberg-Salam theory

$$(g^{1/4} f^{\omega\tau})_{,\omega} - \sqrt{-2\Lambda_b} g^{1/4} [f^{\omega\tau}, \mathcal{A}_\omega] = 4\pi g^{1/4} j^\tau \quad (6)$$

- This determines the value of  $\Lambda_b$  and  $\Lambda_z$ ,

$$-\Lambda_z \approx \Lambda_b = \frac{\alpha}{8l_P^2 \sin^2 \theta_w} \sim 1/l_P^2, \quad (7)$$

and this is consistent with  $\Lambda_z$  being caused by zero-point fluctuations

- Other field equations have tiny extra terms

$$f_{\nu\mu} = 2\mathcal{A}_{[\mu,\nu]} + \sqrt{-2\Lambda_b} [\mathcal{A}_\nu, \mathcal{A}_\mu] + \underline{(f^2)\Lambda_b^{-1/2} + (f'')\Lambda_b^{-1} \dots} \quad (8)$$

$$G_{\nu\mu} = 8\pi T_{\nu\mu} + tr \left( f^\sigma ({}_\nu f_\mu)_\sigma - \frac{1}{4} g_{\nu\mu} f^{\rho\sigma} f_{\sigma\rho} \right) + \Lambda g_{\nu\mu} + \underline{(f^3)\Lambda_b^{-1/2} + (f'f')\Lambda_b^{-1} \dots} \quad (9)$$

## Non-Abelian LRES theory $\approx$ Einstein-Weinberg-Salam theory

- Extra terms in the field equations are  $< 10^{-13}$  of usual terms for worst-case  $|f_{\mu\nu}|$ ,  $|f_{\mu\nu;\alpha}|$  and  $|f_{\mu\nu;\alpha;\beta}|$  accessible to measurement.
- Abelian LRES theory closely approximates Einstein-Maxwell theory: see Gen. Rel. Grav. (Online First), gr-qc/0801.2307.
- Usual Lorentz-force equation results from divergence of Einstein equations
- Exact solutions:
  - EM plane-wave solution is identical to that of Einstein-Maxwell theory.
  - Charged solution and Reissner-Nordström sol. have tiny fractional difference:  
 $10^{-76}$  for  $r=Q=M=M_{\odot}$ ,  $10^{-64}$  for  $r=10^{-17}cm, Q=e, M=M_e$ .

Standard tests	fractional difference from Einstein-Maxwell result	
test case $\rightarrow$	$Q=M=M_{\odot}, r=4M$	$Q=e, M=M_P, r=a_0$
periastron advance	$10^{-78}$	$10^{-91}$
deflection of light	$10^{-79}$	$10^{-57}$
time delay of light	$10^{-78}$	$10^{-56}$

- Possible Proca field ghost with  $M_{Proca}/\hbar = \sqrt{2\Lambda_b} \sim 1/l_P$ , but probably not

**Lagrangian has  $U(1) \otimes SU(2)$  invariance like Weinberg-Salam theory**

- $\mathcal{L} \rightarrow \mathcal{L}$  under  $SU(2)$  gauge transformation, with  $2 \times 2$  matrix  $U$

$$\mathcal{A}_\nu \rightarrow U \mathcal{A}_\nu U^{-1} + \frac{i}{\sqrt{2\Lambda_b}} U_{,\nu} U^{-1}, \quad (10)$$

$$\Rightarrow \hat{\Gamma}_{\nu\mu}^\alpha \rightarrow U \hat{\Gamma}_{\nu\mu}^\alpha U^{-1} + 2\delta_{[\nu}^\alpha U_{,\mu]} U^{-1}, \quad (11)$$

$$\Rightarrow \hat{\mathcal{R}}_{\nu\mu} \rightarrow U \hat{\mathcal{R}}_{\nu\mu} U^{-1} \quad (12)$$

$$N_{\nu\mu} \rightarrow U N_{\nu\mu} U^{-1}, \quad \mathfrak{g}_{\nu\mu} \rightarrow U \mathfrak{g}_{\nu\mu} U^{-1}, \quad f_{\nu\mu} \rightarrow U f_{\nu\mu} U^{-1}. \quad (13)$$

- $\mathcal{L} \rightarrow \mathcal{L}$  under  $U(1)$  gauge transformation, with scalar  $\varphi$

$$\mathcal{A}_\nu \rightarrow \mathcal{A}_\nu + \frac{1}{\sqrt{2\Lambda_b}} \varphi_{,\nu}, \quad (14)$$

$$\Rightarrow \hat{\Gamma}_{\nu\mu}^\alpha \rightarrow \hat{\Gamma}_{\nu\mu}^\alpha - 2iI \delta_{[\nu}^\alpha \varphi_{,\mu]}, \quad (15)$$

$$\Rightarrow \hat{\mathcal{R}}_{\nu\mu} \rightarrow \hat{\mathcal{R}}_{\nu\mu} \quad (16)$$

$$N_{\nu\mu} \rightarrow N_{\nu\mu}, \quad \mathfrak{g}_{\nu\mu} \rightarrow \mathfrak{g}_{\nu\mu}, \quad f_{\nu\mu} \rightarrow f_{\nu\mu}. \quad (17)$$

- $\mathcal{L}^* = \mathcal{L}$  when  $\mathcal{A}_\nu$  and  $f_{\nu\mu}$  are Hermitian

$$\hat{\Gamma}_{\nu\mu}^{\alpha*} = \hat{\Gamma}_{\mu\nu}^{\alpha T}, \quad \hat{\mathcal{R}}_{\nu\mu}^* = \hat{\mathcal{R}}_{\mu\nu}^T, \quad N_{\nu\mu}^* = N_{\mu\nu}^T, \quad N^* = N, \quad (18)$$

$$\Rightarrow \mathcal{A}_\nu^* = \mathcal{A}_\nu^T, \quad f_{\nu\mu}^* = f_{\nu\mu}^T, \quad \mathfrak{g}_{\nu\mu}^* = \mathfrak{g}_{\nu\mu}^T, \quad \mathfrak{g}^* = \mathfrak{g}. \quad (19)$$

## Summary of non-Abelian $\Lambda$ -renormalized Einstein-Schrödinger theory

- $\left( \begin{array}{c} \text{non-Abelian} \\ \text{LRES theory} \end{array} \right) \approx \left( \begin{array}{c} \text{Einstein-Weinberg-} \\ \text{Salam theory} \end{array} \right)$  with  $|\Lambda_z| \approx \frac{\alpha}{8 l_P^2 \sin^2 \theta_w} \sim \frac{1}{l_P^2}$ .
- Extra terms in Weinberg-Salam field equations are  $< 10^{-13}$  of usual terms.
- $\lim_{|\Lambda_z| \rightarrow \infty} \left( \begin{array}{c} \text{LRES} \\ \text{theory} \end{array} \right) = \left( \begin{array}{c} \text{Einstein-Maxwell} \\ \text{theory} \end{array} \right) \quad \left( \begin{array}{c} \text{Gen.Rel.Grav. (Online First)} \\ \text{gr-qc/0801.2307} \end{array} \right)$
- Lagrangian has  $U(1) \otimes SU(2)$  invariance like Weinberg-Salam theory.
- $\mathcal{L}_m$  contains  $\psi_{ev}, \phi$  fields, and could also include rest of Standard Model.
- It's the ES theory but with Hermitian matrix components and a  $\Lambda_z$  term.
- Suggests untried approaches to a complete unified field theory:
  - Larger matrices:  $5 \times 5$  matrices for  $SU(5)$  or  $U(1) \otimes SU(5)$ ?
- For details see gr-qc/0804.1962

Backup charts

## The non-Abelian Ricci tensor

- We use one of many nonsymmetric generalizations of Ricci tensor

$$\hat{\mathcal{R}}_{\nu\mu} = \hat{\Gamma}_{\nu\mu,\alpha}^{\alpha} - \hat{\Gamma}_{(\alpha(\nu),\mu)}^{\alpha} + \frac{1}{2}\hat{\Gamma}_{\nu\mu}^{\sigma}\hat{\Gamma}_{(\sigma\alpha)}^{\alpha} + \frac{1}{2}\hat{\Gamma}_{(\sigma\alpha)}^{\alpha}\hat{\Gamma}_{\nu\mu}^{\sigma} - \hat{\Gamma}_{\nu\alpha}^{\sigma}\hat{\Gamma}_{\sigma\mu}^{\alpha} - \hat{\Gamma}_{[\tau\nu]}^{\tau}\hat{\Gamma}_{[\rho\mu]}^{\rho}/3 \quad (20)$$

- Because it has important invariance properties

$$R_{\rho\tau}(\hat{\Gamma}_{\nu\mu}^{\alpha T}) = R_{\tau\rho}^T(\hat{\Gamma}_{\mu\nu}^{\alpha}) \quad (\text{transposition symmetric}) \quad (21)$$

$$R_{\rho\tau}(U\hat{\Gamma}_{\nu\mu}^{\alpha}U^{-1} + 2\delta_{[\nu}^{\alpha}U_{,\mu]})U^{-1} = UR_{\rho\tau}(\hat{\Gamma}_{\nu\mu}^{\alpha})U^{-1} \quad (\text{almost SU(2) invariant}) \quad (22)$$

$$R_{\rho\tau}(\hat{\Gamma}_{\nu\mu}^{\alpha} - 2iI\delta_{[\nu}^{\alpha}\varphi_{,\mu]}) = R_{\rho\tau}(\hat{\Gamma}_{\nu\mu}^{\alpha}) \quad (\text{U(1) invariant}) \quad (23)$$

- For Abelian fields the third and fourth terms are the same.
- Reduces to the ordinary Ricci tensor for  $\hat{\Gamma}_{[\nu\mu]}^{\alpha} = 0$  and  $\hat{\Gamma}_{\alpha[\nu,\mu]}^{\alpha} = 0$ , as occurs in ordinary general relativity.

## The Lagrangian Density Again

- $\mathcal{A}_\nu$  is defined by

$$\mathcal{A}_\nu = \widehat{\Gamma}_{[\nu\rho]}^\rho / \sqrt{-18\Lambda_b}. \quad (24)$$

- $\widehat{\Gamma}_{\nu\mu}^\alpha$  can be decomposed into  $\tilde{\Gamma}_{\nu\mu}^\alpha$  with the symmetry  $\tilde{\Gamma}_{\nu\alpha}^\alpha = \tilde{\Gamma}_{\alpha\nu}^\alpha$ , and  $\mathcal{A}_\nu$ ,

$$\tilde{\Gamma}_{\nu\mu}^\alpha = \widehat{\Gamma}_{\nu\mu}^\alpha + (\delta_\mu^\alpha \widehat{\Gamma}_{[\sigma\nu]}^\sigma - \delta_\nu^\alpha \widehat{\Gamma}_{[\sigma\mu]}^\sigma) / 3 \quad \Rightarrow \quad \widehat{\Gamma}_{\nu\mu}^\alpha = \tilde{\Gamma}_{\nu\mu}^\alpha + 2\delta_{[\mu}^\alpha \mathcal{A}_{\nu]} \sqrt{-2\Lambda_b}. \quad (25)$$

- The Lagrangian density (3) in terms of  $\mathcal{A}_\mu$ ,  $\tilde{\Gamma}_{\nu\mu}^\alpha$  and  $\tilde{\mathcal{R}}_{\nu\mu} = \mathcal{R}_{\nu\mu}(\tilde{\Gamma})$  is,

$$\begin{aligned} \mathcal{L}(\widehat{\Gamma}_{\rho\tau}^\lambda, N_{\rho\tau}) = & -\frac{1}{16\pi} N^{1/4} \left[ \text{tr}(N^{-1\mu\nu} (\tilde{\mathcal{R}}_{\nu\mu} + 2\mathcal{A}_{[\nu,\mu]} \sqrt{-2\Lambda_b}) + 2\Lambda_b [\mathcal{A}_\nu, \mathcal{A}_\mu]) + 4\Lambda_b \right] \\ & -\frac{1}{4\pi} \Lambda_z g^{1/4} + \mathcal{L}_m(\mathcal{A}_\nu, g_{\mu\nu}, \psi_{e\nu}, \phi, \dots). \end{aligned} \quad (26)$$

- The nonsymmetric Ricci tensor (20) reduces to

$$\tilde{\mathcal{R}}_{\nu\mu} = \tilde{\Gamma}_{\nu\mu,\alpha}^\alpha - \tilde{\Gamma}_{\alpha(\nu,\mu)}^\alpha + \frac{1}{2} \tilde{\Gamma}_{\nu\mu}^\sigma \tilde{\Gamma}_{\sigma\alpha}^\alpha + \frac{1}{2} \tilde{\Gamma}_{\sigma\alpha}^\alpha \tilde{\Gamma}_{\nu\mu}^\sigma - \tilde{\Gamma}_{\nu\alpha}^\sigma \tilde{\Gamma}_{\sigma\mu}^\alpha \quad (27)$$

- We assume the special case  $g_{\nu\mu} = \text{Itr}(g_{\nu\mu})/2$  and  $\tilde{\Gamma}_{\nu\mu}^\alpha = \text{Itr}(\tilde{\Gamma}_{\nu\mu}^\alpha)/2$ .

## The Einstein Equations

- $g_{\mu\nu}$  and  $f_{\mu\nu}$  are defined by (with  $c=G=1$ )

$$g^{1/4} g^{\nu\mu} = N^{1/4} N^{-1(\mu\nu)} \quad (28)$$

$$g^{1/4} f^{\nu\mu} = i N^{1/4} N^{-1[\nu\mu]} \Lambda_b^{1/2} / \sqrt{2} \quad (29)$$

Inverting these definitions gives (after some effort)

$$N_{(\nu\mu)} = g_{\nu\mu} - 2 \left( f^\sigma ({}_{\nu}f_{\mu})_\sigma - \frac{1}{4} g_{\nu\mu} \text{tr}(f^\rho {}_\sigma f^\sigma {}_\rho) / 2 \right) \Lambda_b^{-1} + (f^3) \Lambda_b^{-3/2} \dots \quad (30)$$

$$N_{[\nu\mu]} = f_{\nu\mu} \sqrt{2} i \Lambda_b^{-1/2} + (f^2) \Lambda_b^{-1} \dots \quad (31)$$

- $f_{\mu\nu}$  comes from  $\delta\mathcal{L}/\delta(\sqrt{-N} N^{-1[\mu\nu]}) = 0$  and  $\tilde{\mathcal{R}}_{[\nu\mu]} = (f'') \Lambda_b^{-1/2} \dots$  from (43),

$$N_{[\nu\mu]} = 2\mathcal{A}_{[\mu,\nu]} \sqrt{-2} \Lambda_b^{-1/2} - 2[\mathcal{A}_\nu, \mathcal{A}_\mu] - \tilde{\mathcal{R}}_{[\nu\mu]} \Lambda_b^{-1} \quad (32)$$

$$\Rightarrow f_{\nu\mu} = \mathcal{A}_{\mu,\nu} - \mathcal{A}_{\nu,\mu} + \sqrt{-2} \Lambda_b [\mathcal{A}_\nu, \mathcal{A}_\mu] + (f^2) \Lambda_b^{-1/2} + (f'') \Lambda_b^{-1} \dots \quad (33)$$

- Einstein equations come from  $\delta\mathcal{L}/\delta(\sqrt{-N} N^{-1(\mu\nu)}) = 0$ ,

$$\tilde{G}_{\nu\mu} = 8\pi T_{\nu\mu} - \Lambda_b \text{tr} \left( N_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} N^\rho{}_\rho \right) + \Lambda_b g_{\nu\mu} \quad (34)$$

$$= 8\pi T_{\nu\mu} + \text{tr} \left( f^\sigma ({}_{\nu}f_{\mu})_\sigma - \frac{1}{4} g_{\nu\mu} f^{\rho\sigma} f_{\sigma\rho} \right) + \Lambda_b g_{\nu\mu} + (f^3) \Lambda_b^{-1/2} + (f' f') \Lambda_b^{-1} \dots \quad (35)$$

## Weinberg-Salam equivalent of Ampere's Law

- Maxwell's equations come from  $\delta\mathcal{L}/\delta\mathcal{A}_\tau=0$ ,

$$(g^{1/4} f^{\omega\tau})_{,\omega} - \sqrt{-2\Lambda_b} g^{1/4} [f^{\omega\tau}, \mathcal{A}_\omega] = 4\pi g^{1/4} j^\tau, \quad (36)$$

where  $f_{\nu\mu} \approx 2\mathcal{A}_{[\mu,\nu]} + \sqrt{-2\Lambda_b} [\mathcal{A}_\nu, \mathcal{A}_\mu]$  and

$$j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta\mathcal{L}_m}{\delta\mathcal{A}_\tau}. \quad (37)$$

- $\mathcal{L}_m$  contains  $\psi_{ev}, \phi$  fields of Weinberg-Salam theory,

$$j^\tau = Q \bar{\psi}_{ev} \gamma^\tau \psi_{ev}. \quad (38)$$

## The Connection Equations

- Relation between  $\tilde{\Gamma}_{\mu\nu}^\alpha$  and  $N_{\mu\nu}$  like  $(\sqrt{-g}g^{\tau\rho})_{;\beta} = 0$  comes from  $\delta\mathcal{L}/\delta\tilde{\Gamma}_{\tau\rho}^\beta = 0$ ,

$$\begin{aligned} tr[(N^{1/4}N^{-1\rho\tau})_{,\beta} + \tilde{\Gamma}_{\sigma\beta}^\tau N^{1/4}N^{-1\rho\sigma} + \tilde{\Gamma}_{\beta\sigma}^\rho N^{1/4}N^{-1\sigma\tau} - \tilde{\Gamma}_{\beta\alpha}^\alpha N^{1/4}N^{-1\rho\tau}] \\ = \frac{8\pi\sqrt{2}i}{3}g^{1/4} tr[j^{[\rho]}\delta_\beta^{\tau}]\Lambda_b^{-1/2}. \end{aligned} \quad (39)$$

- Solving these equations gives

$$\tilde{\Gamma}_{(\nu\mu)}^\alpha = \frac{I}{2}g^{\alpha\rho}(g_{\mu\rho,\nu} + g_{\rho\nu,\mu} - g_{\nu\mu,\rho}) + (f'f)\Lambda_b^{-1} \dots, \quad (40)$$

$$\tilde{\Gamma}_{[\nu\mu]}^\alpha = (f')\Lambda_b^{-1} \dots, \quad (41)$$

$$\tilde{\mathcal{R}}_{(\nu\mu)} = R_{\nu\mu} + (f'f')\Lambda_b^{-1} \dots, \quad (42)$$

$$\tilde{\mathcal{R}}_{[\nu\mu]} = (f'')\Lambda_b^{-1/2} \dots. \quad (43)$$

$\Rightarrow \tilde{\mathcal{R}}_{(\nu\mu)} \approx R_{\nu\mu}$  and  $f_{\nu\mu} \approx 2\mathcal{A}_{[\mu,\nu]} + \sqrt{-2\Lambda_b} [\mathcal{A}_\nu, \mathcal{A}_\mu]$  with fractional differences  $< 10^{-13}$  for worst-case  $|f_{\mu\nu}|$ ,  $|f_{\mu\nu;\alpha}|$ ,  $|f_{\mu\nu;\alpha;\beta}|$  accessible to measurement (e.g.  $10^{20}eV$ ,  $10^{34}Hz$   $\gamma$ -rays).

## Proca waves as Pauli-Villars ghosts?

- If wave-packet Proca waves exist and if they have negative energy, perhaps the Proca field functions as a built-in Pauli-Villars ghost

$$\omega_c = \omega_{Proca} = \sqrt{2\Lambda_b} \ , \quad -\Lambda_z \approx \Lambda_b = \frac{\alpha}{8l_P^2 \sin^2 \theta_w} \quad (44)$$

$$\Lambda_z = -\frac{\omega_c^4 l_P^2}{2\pi} \left( \begin{array}{c} \text{fermion} \\ \text{spin states} \end{array} - \begin{array}{c} \text{boson} \\ \text{spin states} \end{array} \right) \quad (45)$$

$$\Rightarrow \left( \begin{array}{c} \text{fermion} \\ \text{spin states} \end{array} - \begin{array}{c} \text{boson} \\ \text{spin states} \end{array} \right) = \frac{4\pi \sin^2 \theta_w}{\alpha} = 412.8 \pm 2 \quad (46)$$

- In this case  $4\pi \sin^2 \theta_w / \alpha$  or its “bare” value at  $\omega_c$  should be an integer.
- For the Standard Model the difference in (46) is about 60.
- Non-Abelian LRES theory works for  $d \times d$  instead of  $2 \times 2$  matrices.
- Perhaps some value of “d” is consistent with (46).
- $SU(5)$  almost unifies the Standard Model, how about  $U(1) \otimes SU(5)$  ?