Λ-renormalized Einstein-Schrödinger theory: an alternative to Einstein-Maxwell theory

James A. Shifflett
Department of Physics
WUGRAV Gravity Group
Washington University in St. Louis

shifflet@hbar.wustl.edu

Supported in part by NSF grant PHY 03-53180.

APS Meeting, Jacksonville, Florida
14-17 Apr, 2007

Λ-renormalized Einstein-Schrödinger (LRES) theory

- Einstein-Maxwell theory can be derived from a Palatini Lagrangian density,

$$\mathcal{L}(\Gamma^\lambda_{\rho\tau}, g_{\rho\tau}, A_\nu) = -\frac{1}{16\pi}\left[\sqrt{-g}\, g^{\mu\nu} R_{\nu\mu}(\Gamma) + 2\Lambda\sqrt{-g}\,\right] + \frac{1}{4\pi}\sqrt{-g} A_{[\alpha,\beta]} g^{\alpha\mu} g^{\beta\nu} A_{[\mu,\nu]} + \mathcal{L}_m(u^\nu, \psi_e, A_\nu, g_{\mu\nu}, \cdots). \quad (1)$$

- LRES theory uses nonsymmetric $\widehat{\Gamma}^\alpha_{\mu\nu}$ and $N_{\mu\nu}$, excludes $\sqrt{-g}A_{[\alpha,\beta]}g^{\alpha\mu}g^{\beta\nu}A_{[\mu,\nu]}$, and includes Λ_z from zero-point fluctuations,

$$\mathcal{L}(\widehat{\Gamma}^\lambda_{\rho\tau}, N_{\rho\tau}) = -\frac{1}{16\pi}\left[\sqrt{-N} N^{-1\mu\nu} \mathcal{R}_{\nu\mu}(\widehat{\Gamma}\,) + 2\Lambda_b\sqrt{-N}\,\right] - \frac{1}{16\pi} 2\Lambda_z\sqrt{-g} + \mathcal{L}_m(u^\nu, \psi_e, A_\nu, g_{\mu\nu}, \cdots), \qquad N = \det(N_{\mu\nu}) \quad (2)$$

where the "bare" $\Lambda_b \approx -\Lambda_z$ so the "physical" $\Lambda = \Lambda_b + \Lambda_z$ matches measurement, and the metric $g_{\mu\nu}$ and potential A_ν are defined by

$$\sqrt{-g}g^{\nu\mu} = \sqrt{-N}N^{-1(\mu\nu)}, \qquad A_\nu = \widehat{\Gamma}^\rho_{[\nu\rho]}/\sqrt{-18\Lambda_b}\,, \qquad (\text{with } c = G = 1). \quad (3)$$

- $$\lim_{|\Lambda_z|\to\infty}\binom{\text{LRES}}{\text{theory}} = \binom{\text{Einstein}-\text{Maxwell}}{\text{theory}} \quad \text{but} \quad \omega_c \sim \frac{1}{l_P} \;\Rightarrow\; |\Lambda_z| \sim \omega_c^4 l_P^2 \sim \frac{1}{l_P^2}\,.$$

LRES theory avoids the problems of Einstein-Schrödinger theory

• Matches measurement as well as Einstein-Maxwell theory.

• Definitely predicts a Lorentz force:
- Usual Lorentz force equation results from divergence of Einstein equations,
- Lorentz force also results from the EIH method, with $\mathcal{L}_m = 0$.

• Avoids ghosts:
- With a cutoff frequency $\omega_c \sim 1/l_P$ we have $\Lambda_z \sim -\omega_c^4 l_P^2$ (with c=G=1),
- Ghosts are cut off because they would have $\omega_{ghost} = \sqrt{-2\Lambda_z} \sim \sqrt{2}\,\omega_c^2 l_P > \omega_c$,
- If we fully renormalize with $\omega_c \rightarrow \infty$ then $\omega_{ghost} \rightarrow \infty$, meaning no ghost.

• Well motivated:
- It's a vacuum energy renormalization of Einstein-Schrödinger theory,
- $\Lambda_z\sqrt{-g}$ term should be expected to occur as a quantization effect,
- Zero-point fluctuations are essential to QED - they cause the Casimir effect,
- $\Lambda = \Lambda_b + \Lambda_z$ is similar to mass/charge/field-strength renormalization in QED,
- $\Lambda_z\sqrt{-g}$ modification has never been considered before.

LRES theory matches measurement as well as Einstein-Maxwell theory

• Reduces to ordinary GR without electromagnetism for symmetric fields.

• Extra terms in Einstein and Maxwell equations are $<10^{-16}$ of usual terms for worst-case $|F_{\mu\nu}|$, $|F_{\mu\nu;\alpha}|$ and $|F_{\mu\nu;\alpha;\beta}|$ accessible to measurement.

• Exact solutions:
- EM plane-wave solution is identical to that of Einstein-Maxwell theory.
- Charged solution and Reissner-Nordström sol. have tiny fractional difference:
10^{-76} for $r=Q=M=M_{\odot}$, 10^{-64} for $r=10^{-17}cm, Q=e, M=M_e$.

• Standard tests	fractional difference from Einstein-Maxwell result	
test case →	extremal charged black hole $Q=M=M_{\odot}, r=4M$	atomic parameters $Q=e, M=M_P, r=a_0$
periastron advance	10^{-78}	10^{-91}
deflection of light	10^{-79}	10^{-57}
time delay of light	10^{-78}	10^{-56}

• Other Standard Model fields can be added just like Einstein-Maxwell theory:
- Energy levels of Hydrogen atom have fractional difference of $<10^{-90}$.

Why pursue LRES theory if it's so close to Einstein-Maxwell theory?

• It unifies gravitation and electromagnetism in a classical sense.

• Quantization of LRES theory is untried approach to quantization of gravity:
- LRES theory gets much different than Einstein-Maxwell theory as $k\rightarrow 1/l_P$,
- This could possibly fix some infinities which spoil the quantization of GR.

• LRES theory suggests untried approaches to a complete unified field theory:
- Higher dimensions, but with LRES theory instead of vacuum GR?
- Non-abelian fields, but with LRES theory instead of Einstein-Maxwell?

• We still don't have a unified field theory, 50 years after Einstein:
- Standard Model: excludes gravity, 25 parameters, not very "beautiful",
- String theory: background dependent, spin-2 particle $\Rightarrow$ GR?, 10^{500} versions, problems accounting for $\Lambda>0$ and broken symmetry, little predictive ability.

Summary of Λ-renormalized Einstein-Schrödinger theory

- $\lim_{|\Lambda_z|\to\infty}\begin{pmatrix}\text{LRES}\\ \text{theory}\end{pmatrix}=\begin{pmatrix}\text{Einstein}-\text{Maxwell}\\ \text{theory}\end{pmatrix}$ but $\omega_c\sim\dfrac{1}{l_P} \Rightarrow |\Lambda_z|\sim\omega_c^4 l_P^2\sim\dfrac{1}{l_P^2}$.
- Matches measurement as well as Einstein-Maxwell theory.
- Reduces to ordinary GR without electromagnetism for symmetric fields.
- Other Standard Model fields can be added just like Einstein-Maxwell theory.
- Avoids the problems of the original Einstein-Schrödinger theory.
- Well motivated - it's the ES theory but with a quantization effect.
- Unifies gravitation and electromagnetism in a classical sense.
- Suggests untried approaches to a complete quantized unified field theory.
- For the details see my papers: www.arxiv.org/abs/gr-qc/0310124, www.arxiv.org/abs/gr-qc/0403052, www.arxiv.org/abs/gr-qc/0411016.

Backup charts

The Lagrangian Density Again

- A_ν and $F_{\mu\nu}$ are defined by (with c=G=1)

$$A_\nu = \widehat{\Gamma}^\rho_{[\nu\rho]}/\sqrt{-18\Lambda_b}, \qquad F_{\mu\nu} = A_{\nu,\,\mu} - A_{\mu,\nu}. \tag{4}$$

- $\widehat{\Gamma}^\alpha_{\nu\mu}$ can be decomposed into $\tilde{\Gamma}^\alpha_{\nu\mu}$ with the symmetry $\tilde{\Gamma}^\alpha_{\nu\alpha} = \tilde{\Gamma}^\alpha_{\alpha\nu}$, and A_ν,

$$\tilde{\Gamma}^\alpha_{\nu\mu} = \widehat{\Gamma}^\alpha_{\nu\mu} + (\delta^\alpha_\mu \widehat{\Gamma}^\sigma_{[\sigma\nu]} - \delta^\alpha_\nu \widehat{\Gamma}^\sigma_{[\sigma\mu]})/3 \quad \Rightarrow \quad \widehat{\Gamma}^\alpha_{\nu\mu} = \tilde{\Gamma}^\alpha_{\nu\mu} + 2\delta^\alpha_{[\mu} A_{\nu]} \sqrt{-2}\, \Lambda_b^{1/2}. \tag{5}$$

- The "Hermitianized Ricci tensor" in (2) reduces to the ordinary Ricci tensor for symmetric fields with $\Gamma^\alpha_{[\nu\mu]} = 0$ and $\Gamma^\alpha_{\alpha[\nu,\mu]} = R^\alpha{}_{\alpha\mu\nu}/2 = 0$,

$$\mathcal{R}_{\nu\mu}(\widehat{\Gamma}\,) = \widehat{\Gamma}^\alpha_{\nu\mu,\alpha} - \widehat{\Gamma}^\alpha_{(\alpha(\nu),\mu)} + \widehat{\Gamma}^\rho_{\nu\mu}\widehat{\Gamma}^\alpha_{(\rho\alpha)} - \widehat{\Gamma}^\rho_{\nu\alpha}\widehat{\Gamma}^\alpha_{\rho\mu} - \widehat{\Gamma}^\tau_{[\tau\nu]}\widehat{\Gamma}^\alpha_{[\alpha\mu]}/3. \tag{6}$$

- $\mathcal{R}_{\nu\mu}$ exhibits both charge conjugation symmetry and gauge invariance

$$\mathcal{R}_{\mu\nu}(\widehat{\Gamma}^T) = \mathcal{R}_{\nu\mu}(\widehat{\Gamma}\,), \qquad \mathcal{R}_{\nu\mu}(\widehat{\Gamma}^\alpha_{\rho\tau} + \delta^\alpha_{[\rho}\phi_{,\tau]}) = \mathcal{R}_{\nu\mu}(\widehat{\Gamma}^\alpha_{\rho\tau}). \tag{7}$$

- The Lagrangian density (2) in terms of A_μ, $\tilde{\Gamma}^\alpha_{\nu\mu}$ and $\tilde{\mathcal{R}}_{\nu\mu} = \mathcal{R}_{\nu\mu}(\tilde{\Gamma})$ is,

$$\begin{aligned}\mathcal{L}(\widehat{\Gamma}^\lambda_{\rho\tau}, N_{\rho\tau}) = &-\frac{1}{16\pi}\left[\sqrt{-N}N^{-1\mu\nu}(\tilde{\mathcal{R}}_{\nu\mu} + 2A_{[\nu,\mu]}\sqrt{-2}\,\Lambda_b^{1/2}) + 2\Lambda_b\sqrt{-N}\,\right] \\ &-\frac{1}{16\pi}2\Lambda_z\sqrt{-g} + \mathcal{L}_m(u^\nu, \psi_e, A_\nu, g_{\mu\nu}, \cdots).\end{aligned} \tag{8}$$

The Einstein Equations

- $g_{\mu\nu}$ and $f_{\mu\nu}$ are defined by (with c=G=1)

$$\sqrt{-g}\, g^{\nu\mu} = \sqrt{-N} N^{-1(\mu\nu)}, \tag{9}$$

$$\sqrt{-g} f^{\nu\mu} = \sqrt{-N} N^{-1[\mu\nu]} \Lambda_b^{1/2}/\sqrt{-2}. \tag{10}$$

Inverting these definitions gives (after some effort)

$$N_{(\nu\mu)} = g_{\nu\mu} - 2\left(f_\nu{}^\alpha f_{\alpha\mu} - \frac{1}{4}\, g_{\nu\mu} f^{\rho\alpha} f_{\alpha\rho}\right)\Lambda_b^{-1} + \mathcal{O}(\Lambda_b^{-2}), \tag{11}$$

$$N_{[\nu\mu]} = f_{\nu\mu}\sqrt{-2}\,\Lambda_b^{-1/2} + \mathcal{O}(\Lambda_b^{-3/2}). \tag{12}$$

- $f_{\mu\nu} \approx F_{\mu\nu}$ comes from $\delta\mathcal{L}/\delta(\sqrt{-N} N^{-1[\mu\nu]}) = 0$ and $\tilde{\mathcal{R}}_{[\nu\mu]} = \mathcal{O}(\Lambda_b^{-1/2})$ from (26),

$$N_{[\nu\mu]} = 2A_{[\mu,\nu]}\sqrt{-2}\,\Lambda_b^{-1/2} - \tilde{\mathcal{R}}_{[\nu\mu]}\Lambda_b^{-1}, \tag{13}$$

$$\Rightarrow \quad f_{\nu\mu} = A_{\mu,\nu} - A_{\nu,\mu} + \mathcal{O}(\Lambda_b^{-1}). \tag{14}$$

- Einstein equations come from $\delta\mathcal{L}/\delta(\sqrt{-N} N^{-1(\mu\nu)}) = 0$,

$$\tilde{\mathcal{R}}_{(\nu\mu)} - \frac{1}{2}\, g_{\nu\mu}\tilde{\mathcal{R}}^\rho_\rho = 8\pi T_{\nu\mu} - \Lambda_b\left(N_{(\nu\mu)} - \frac{1}{2}\, g_{\nu\mu} N^\rho_\rho\right) + \Lambda_z\, g_{\nu\mu} \tag{15}$$

$$= 8\pi T_{\nu\mu} + 2\left(f_\nu{}^\alpha f_{\alpha\mu} - \frac{1}{4}\, g_{\nu\mu} f^{\rho\alpha} f_{\alpha\rho}\right) + \Lambda g_{\nu\mu} + \mathcal{O}(\Lambda_b^{-1}). \tag{16}$$

Maxwell's Equations

- Maxwell's equations come from $\delta\mathcal{L}/\delta A_\tau = 0$ and $F_{\mu\nu} = A_{\nu,\,\mu} - A_{\mu,\nu}$,

$$f^{\nu\tau}{}_{;\,\nu} = 4\pi j^\tau, \quad (17)$$
$$F_{[\mu\nu,\alpha]} = 0, \quad (18)$$

where $f_{\mu\nu} \approx F_{\mu\nu}$ and

$$j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta\mathcal{L}_m}{\delta A_\tau}. \quad (19)$$

- $\mathcal{L}_m$ may contain other fields just like Einstein-Maxwell theory,

$$j^\tau = Q\bar{\psi}_e \gamma^\tau \psi_e \quad \text{for spin}-1/2, \quad (20)$$
$$j^\tau = \rho u^\tau \quad \text{for classical hydrodynamics.} \quad (21)$$

The Connection Equations

- Relation between $\tilde{\Gamma}^\alpha_{\mu\nu}$ and $N_{\mu\nu}$ like $(\sqrt{-g}g^{\tau\rho})_{;\,\beta}\!=\!0$ comes from $\delta\mathcal{L}/\delta\tilde{\Gamma}^\beta_{\tau\rho}\!=\!0$,

$$(\sqrt{-N}N^{-1\rho\tau})_{,\,\beta}+\tilde{\Gamma}^\tau_{\nu\beta}\sqrt{-N}N^{-1\rho\nu}+\tilde{\Gamma}^\rho_{\beta\nu}\sqrt{-N}N^{-1\nu\tau}-\tilde{\Gamma}^\alpha_{\beta\alpha}\sqrt{-N}N^{-1\rho\tau}$$
$$=\frac{8\pi}{3}\sqrt{-g}\,j^{[\rho}\delta^{\tau]}_\beta\sqrt{-2}\,\Lambda_b^{-1/2}. \quad (22)$$

- Solving these equations gives

$$\tilde{\Gamma}^\alpha_{(\nu\mu)} = \frac{1}{2}g^{\alpha\rho}(g_{\mu\rho,\nu}+g_{\rho\nu,\mu}-g_{\nu\mu,\rho})+\mathcal{O}(\Lambda_b^{-1}), \quad (23)$$
$$\tilde{\Gamma}^\alpha_{[\nu\mu]} = \mathcal{O}(\Lambda_b^{-1/2}), \quad (24)$$
$$\tilde{\mathcal{R}}_{(\nu\mu)} = R_{\nu\mu}+(\text{terms like } f^{\alpha\tau}f_{\tau(\mu;\nu);\alpha}\Lambda_b^{-1} \text{ and } f^\rho{}_{\mu;\alpha}f^\alpha{}_{\nu;\rho}\Lambda_b^{-1}), \quad (25)$$
$$\tilde{\mathcal{R}}_{[\nu\mu]} = (\text{terms like } f_{[\mu\nu,\tau];}{}^\tau\Lambda_b^{-1/2},\ f^\tau{}_{[\mu;[\nu];\tau]}\Lambda_b^{-1/2} \text{ and } j_{[\nu,\mu]}\Lambda_b^{-1/2}). \quad (26)$$

$\Rightarrow \tilde{\mathcal{R}}_{(\nu\mu)}\approx R_{\nu\mu}$ and $f_{\nu\mu}\approx F_{\nu\mu}$ with fractional differences $<10^{-16}$ for worst-case $|f_{\mu\nu}|$, $|f_{\mu\nu;\alpha}|$, $|f_{\mu\nu;\alpha;\beta}|$ accessible to measurement (e.g. $10^{20}eV, 10^{34}Hz$ γ-rays).

The Generalized Contracted Bianchi Identity

- A generalized contracted Bianchi identity results from (22),

$$(\sqrt{-N}N^{-1\sigma\nu}\tilde{\mathcal{R}}_{\nu\lambda}+\sqrt{-N}N^{-1\nu\sigma}\tilde{\mathcal{R}}_{\lambda\nu})_{,\,\sigma}-\sqrt{-N}N^{-1\sigma\nu}\tilde{\mathcal{R}}_{\nu\sigma,\,\lambda}=0. \tag{27}$$

- It may also be written in the manifestly covariant form,

$$(\sqrt{-N}N^{-1\sigma\nu}\tilde{\mathcal{R}}_{\nu\lambda}+\sqrt{-N}N^{-1\nu\sigma}\tilde{\mathcal{R}}_{\lambda\nu})_{;\,\sigma}-\sqrt{-N}N^{-1\sigma\nu}\tilde{\mathcal{R}}_{\nu\sigma;\,\lambda}=0, \tag{28}$$

- Or in a third form,

$$\tilde{G}^{\sigma}{}_{\lambda;\,\sigma}=\left(\frac{3}{2}f^{\sigma\nu}\tilde{\mathcal{R}}_{[\sigma\nu,\lambda]}+4\pi j^{\nu}\tilde{\mathcal{R}}_{[\nu\lambda]}\right)\sqrt{-2}\,\Lambda_b^{-1/2}, \tag{29}$$

where

$$\tilde{G}_{\nu\mu}=\tilde{\mathcal{R}}_{(\nu\mu)}-\frac{1}{2}\,g_{\nu\mu}\tilde{\mathcal{R}}^{\rho}_{\rho}. \tag{30}$$

- The usual contracted Bianchi identity $2(\sqrt{-g}g^{\sigma\nu}R_{\nu\lambda})_{,\,\sigma}-\sqrt{-g}g^{\sigma\nu}R_{\nu\sigma,\,\lambda}=0$, or $G^{\sigma}{}_{\lambda;\,\sigma}=0$ is also valid.

The Lorentz Force Equation

• Lorentz force equation comes from divergence of the Einstein equations (15)

$$T^{\nu}_{\mu;\,\nu} = F_{\mu\nu} j^{\nu} \tag{31}$$

where

$$j^{\tau} = \frac{-1}{\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta A_\tau}, \tag{32}$$

$$T_{\mu\nu} = S_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} S^{\alpha}_{\alpha}, \tag{33}$$

$$S_{\mu\nu} = \frac{2\,\delta \mathcal{L}_m}{\delta(\sqrt{-g}g^{\nu\mu})}\,. \tag{34}$$

• Here we have used equations (29,17,13) and the following identity which can be derived using only the definitions of $g_{\mu\nu}$ and $f_{\mu\nu}$,

$$\left(N^{(\mu}{}_{\sigma)} - \frac{1}{2}\,\delta^{\mu}_{\sigma} N^{\rho}_{\rho}\right)_{;\,\mu} = \left(\frac{3}{2} f^{\nu\rho} N_{[\nu\rho,\sigma]} + f^{\nu\rho}{}_{;\nu} N_{[\rho\sigma]}\right)\sqrt{-2}\,\Lambda_b^{-1/2}. \tag{35}$$

• Covariant derivative ";" is always done using the Christoffel connection formed from the symmetric metric $g_{\mu\nu}$.

An Exact Charged Solution

- This charged solution is very close to the Reissner-Nordström solution,

$$g_{\nu\mu} = \breve{c} \begin{pmatrix} a & & & \\ & -1/a\breve{c}^2 & & \\ & & -r^2 & \\ & & & -r^2 \sin^2\theta \end{pmatrix}, \tag{36}$$

$$f_{\nu\mu} = \frac{1}{\breve{c}} \begin{pmatrix} 0 & Q/r^2 & & \\ -Q/r^2 & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \tag{37}$$

$$A_0 = \frac{Q}{r}\left[1 + \frac{M}{\Lambda_b r^3} - \frac{4Q^2}{5\Lambda_b r^4} + \mathcal{O}(\Lambda_b^{-2})\right], \tag{38}$$

where

$$a = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}\left[1 + \frac{Q^2}{10\Lambda_b r^4} + \mathcal{O}(\Lambda_b^{-2})\right], \qquad \breve{c} = \sqrt{1 - \frac{2Q^2}{\Lambda_b r^4}}. \tag{39}$$

- Additional terms are tiny for worst-case radii accessible to measurement:
  - $Q^2/\Lambda_b r^4 \sim 10^{-76} @ r = Q = M = M_\odot$; $\sim 10^{-64} @ r = 10^{-17} cm, Q = e, M = M_e$,
  - $M/\Lambda_b r^3 \sim 10^{-76} @ r = Q = M = M_\odot$; $\sim 10^{-70} @ r = 10^{-17} cm, Q = e, M = M_e$.